

# Concurrent Game Structures with Roles\*

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In the following paper we present a new semantics for the well-known strategic logic ATL. It is based on adding *roles* to concurrent game structures, that is at every state, each agent belongs to exactly one role, and the role specifies what actions are available to him at that state. We show advantages of the new semantics, provide motivating examples based on sensor networks, and analyze model checking complexity.

## 1 Introduction

ATL [1] is not only a highly-expressive and powerful strategic logic, but also has a relatively low (polynomial) model checking complexity. However, as investigated by Jamroga and Dix [5], in order for the complexity to be polynomial, the number of agents must be *fixed*. If the number of agents is taken as a parameter, model checking ATL is  $\Delta_2^P$ -complete or  $\Delta_3^P$ -complete depending on model representation [6]. Also, van der Hoek, Lomuscio and Wooldridge show in [3] that the complexity of model checking is polynomial only if an *explicit* enumeration of *all* components of the model is assumed. For models represented in *reactive modules language* (RML) complexity of model checking for ATL becomes as hard as the satisfiability problem for this logic, namely EXPTIME [3].

We present an alternative semantics that interprets formulas of ordinary ATL over concurrent game structures with *roles*. Such structures introduce an extra element – a set  $R$  of roles and associates each agent with exactly one role which are considered *homogeneous* in the sense that all consequences of the actions of the agents belonging to the topical role is captured by considering only the number of “votes” an action gets (one vote per agent).

We present the revised formalism for ATL in Section 2, discuss model checking results in Section 3 and conclude in Section 4.

## 2 Role-based semantics for ATL

In this section we will introduce *concurrent game structures with roles* (RCGS), illustrate them with an example and show in Theorem 1 that treating RCGS or CGS as the semantics of ATL are equivalent.

We will very often refer to sets of natural numbers from 1 to some number  $n \geq 1$ . To simplify the reference to such sets we introduce the notation  $[n] = \{1, \dots, n\}$ . Furthermore we will let  $A^B$  denote the set of functions from  $B$  to  $A$ . We will often also work with tuples  $v = \langle v_1, \dots, v_n \rangle$  and view  $v$  as a function with domain  $[n]$  and write  $v(i)$  for  $v_i$ . Given a function  $f : A \times B \rightarrow C$  and  $a \in A$ , we will use  $f_a$  to denote the function  $B \rightarrow C$  defined by  $f_a(b) = f(a, b)$  for all  $b \in B$ .

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**Definition 1.** An RCGS is a tuple  $H = \langle \mathcal{A}, R, \mathcal{R}, Q, \Pi, \pi, \mathbb{A}, \delta \rangle$  where:

- $\mathcal{A}$  is a non-empty set of players. In this text we assume  $\mathcal{A} = [n]$  for some  $n \in \mathbb{N}$ , and we reserve  $n$  to mean the number of agents.
- $Q$  is the non-empty set of states.
- $R$  is a non-empty set of roles. In this text we assume  $R = [i]$  for some  $i \in \mathbb{N}$ .
- $\mathcal{R} : Q \times \mathcal{A} \rightarrow R$ . For a coalition  $A$  we write  $A_{r,q}$  to denote the agents in  $A$  which belong to role  $r$  at  $q$ , and notably  $\mathcal{A}_{r,q}$  are all the agents in role  $r$  at  $q$ .
- $\Pi$  is a set of propositional letters and  $\pi : Q \rightarrow \wp(\Pi)$  maps each state to the set of propositions true in it.
- $\mathbb{A} : Q \times R \rightarrow \mathbb{N}^+$  is the number of available actions in a given state for a given role.
- For  $\mathcal{A} = [n]$ , we say that the set of complete votes for a role  $r$  in a state  $q$  is  $V_r(q) = \{v_{r,q} \in [n]^{\mathbb{A}(q,r)} \mid \sum_{1 \leq a \leq \mathbb{A}(q,r)} v_{r,q}(a) = |\mathcal{A}_{r,q}|\}$ , the set of functions from the available actions to the number of agents performing the action. The functions in this set account for the actions of all the agents. The set of complete profiles at  $q$  is  $P(q) = \prod_{r \in R} V_r(q)$ . For each  $q \in Q$  we have a transition function at  $q$ ,  $\delta_q : P(q) \rightarrow Q$  defining a partial function  $\delta : Q \times \bigcup_{q \in Q} P(q) \rightarrow Q$  such that for all  $q \in Q, P \in P(q)$ ,  $\delta(q, P) = \delta_q(P)$ .

The following example illustrates how RCGS differs from an ordinary concurrent game structure:

**Example 1** (Sensor networks). A wireless sensor network is a system composed of a number of (homogeneous) sensors that can be triggered by various stimuli. In Figure 1 we show a 1-tier (i.e., completely homogeneous) sensor network with  $n$  sensors. There are two states in the system with labels corresponding to an indicator of the network.  $\neg p$  stands for idle state of the network, while  $p$  indicates that the network detected a stimulus. In this very simple example we say that  $k$  is our threshold, i.e. if at least  $k$  number of sensors detect something, then  $p$ . Since all the sensors behave in the same way we say the role of sensors is homogeneous. Hence the system can be modeled using only a single role. This gives us the model depicted in Figure 1. One can easily add another role to the model if needed, for example in a scenario with a “controller” who processes the reported signals, or in a 2-tier network with several types of sensors.

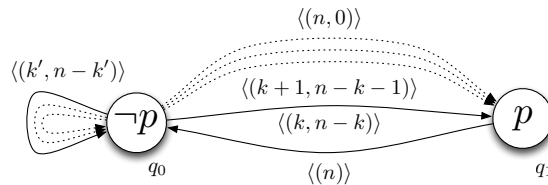


Figure 1: A depiction of  $H_1$  – a simple 1-tier sensor network.

A more complex example is presented in Figure 2, where we add another role to our structure, that of a supervisor or controller. The supervisor can act upon sensors' actions, i.e. if the sensors report that  $p$ , the supervisor can perform  $q$ . As illustrated by the drawing, the supervisor has three actions available: he can wait, he can reject the message or he can accept the message and proceed to state  $q_2$  performing  $q$  (e.g., call the police in an intrusion detection scenario). Finally, in Figure 3 we sketch

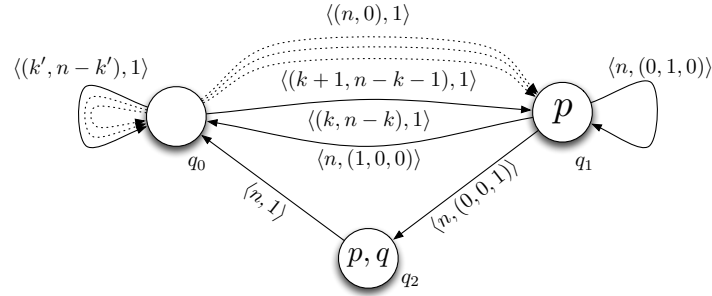


Figure 2: A sketch of structure  $H_2$ : a 1-tier sensor network with a supervisor.

a multi-tier example, with two different types of sensors,  $n_1$  and  $n_2$ , each type with its corresponding role. The transition function with the addition of a new role looks like this:

$$\delta_{q_0}(\langle (x_1, n_1 - x_1), (x_2, n_2 - x_2) \rangle) = \begin{cases} q_1, & x_1 \geq t_1 \wedge x_2 \geq t_2 \\ q_0, & \text{otherwise} \end{cases}$$

$$\delta_{q_1}(\langle n_1, n_2 \rangle) = q_0$$

where  $t_1, t_2$  are thresholds set according to significance of the sensors.

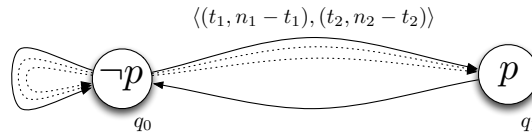


Figure 3: A sketch of structure  $H_3$ : a multi-tier sensor network example.

These simple structures show the benefit of using roles when modelling scenarios which involve a high degree of homogeneity among agents. In this simplified sensor setting a sensor either signals that he has made a relevant observation or he does not – a binary choice. If modelled using concurrent game structures without roles, models would have  $2^n$  number of possible action profiles in state  $q_0$ , since the identity of the agents signaling that they have made an observation has to be accounted for. This, however, is irrelevant for the high-level protocol – all that matters is how many sensors of a given type signal that they have made an observation. With roles we can exploit this, and we only need to account for the genuinely different scenarios that can occur – corresponding to the number of sensors of each type that decide to signal that they have made an observation. In the case of just a single role, this means that we get  $n$  as opposed to  $2^n$  number of different profiles, and the size of the model goes from exponential to linear in the number of sensors. In general, as we will show in Section 3, we shift the exponential dependence in the size of models from the number of agents to the number of roles.

Given a role  $r$ , a state  $q$  and a coalition  $A$ , the set of  $A$ -votes for  $r$  at  $q$  is  $V_r(q, A)$ , defined as:

$$V_r(q, A) = \left\{ v \in [|A_{r,q}|]^{|A(q,r)|} \mid \sum_{a \in [A(q,r)]} v(a) = |A_{r,q}| \right\}.$$

The  $A$ -votes for  $r$  at  $q$  give the possible ways agents in  $A$  that are in role  $r$  at  $q$  can vote. Given a state  $q$  and a coalition  $A$ , we define the set of  $A$ -profiles at  $q$ :

$$P(q, A) = \{\langle v_1, \dots, v_{|R|} \rangle \mid 1 \leq i \leq |R| : v_i \in V_r(q, A)\}.$$

For any  $v \in V_r(q, A)$  and  $w \in V_r(q, B)$  we write  $v \leq w$  iff for all  $i \in [\mathbb{A}(q, r)]$  we have  $v(i) \leq w(i)$ . If  $v \leq w$ , we say that  $w$  *extends*  $v$ . If  $F = \langle v_1, \dots, v_R \rangle \in P(q, A)$  and  $F' = \langle v'_1, \dots, v'_R \rangle \in P(q, B)$  with  $v_i \leq v'_i$  for every  $1 \leq i \leq |R|$ , we say that  $F \leq F'$  and that  $F$  *extends*  $F'$ . Given a (partial) profile  $F'$  at a state  $q$  we write  $\text{ext}(q, F')$  for the set of all complete profiles that extend  $F'$ .

Given two states  $q, q' \in Q$ , we say that  $q'$  is a *successor* of  $q$  if there is some  $F \in P(q)$  such that  $\delta(q, F) = q'$ . A *computation* is an infinite sequence  $\lambda = q_0 q_1 \dots$  of states such that for all positions  $i \geq 0$ ,  $q_{i+1}$  is a successor of  $q_i$ . We follow standard abbreviations, hence a  $q$ -computation denotes a computation starting at  $q$ , and  $\lambda[i]$ ,  $\lambda[0, i]$  and  $\lambda[i, \infty]$  denote the  $i$ -th state, the finite prefix  $q_0 q_1 \dots q_i$  and the infinite suffix  $q_i q_{i+1} \dots$  of  $\lambda$  for any computation  $\lambda$  and its position  $i \geq 0$ , respectively. An  $A$ -*strategy* for  $A \subseteq \mathcal{A}$  is a function  $s_A : Q \rightarrow \bigcup_{q \in Q} P(q, A)$  such that  $s_A(q) \in P(q, A)$  for all  $q \in Q$ . That is,  $s_A$  maps states to  $A$ -profiles at that state. The set of all  $A$ -strategies is denoted by  $\text{strat}(A)$ . When needed to distinguish between different structures we write  $\text{strat}(S, A)$  to indicate that we are talking about the set of strategies for  $A$  in structure  $S$ . If  $s$  is an  $\mathcal{A}$ -strategy and we apply  $\delta_q$  to  $s(q)$ , we obtain a unique new state  $q' = \delta_q(s(q))$ . Iterating, we get the *induced* computation  $\lambda_{s, q} = q_0 q_1 \dots$  such that  $q = q_0$  and  $\forall i \geq 0 : \delta_{q_i}(s(q_i)) = q_{i+1}$ . Given two strategies  $s$  and  $s'$ , we say that  $s \leq s'$  iff  $\forall q \in Q : s(q) \leq s'(q)$ . Given an  $A$ -strategy  $s_A$  and a state  $q$  we get an associated *set* of computations  $\text{out}(s_A, q)$ . This is the set of all computations that can result when at any state, the players in  $A$  are voting/acting in the way specified by  $s_A$ , that is  $\text{out}(s_A, q) = \{\lambda_{s, q} \mid s \text{ is an } \mathcal{A}\text{-strategy and } s \geq s_A\}$ .

Given the definitions above, we can interpret ATL formulas in the following manner, leaving out the propositional cases and abbreviations:

**Definition 2.** Given a RCGS  $S$  and a state  $q$  in  $S$ , we define the satisfaction relation  $\models$  inductively:

- $S, q \models \langle\langle A \rangle\rangle \bigcirc \phi$  iff there is  $s_A \in \text{strat}(A)$  such that for all  $\lambda \in \text{out}(s_A, q)$ , we have  $S, \lambda[1] \models \phi$
- $S, q \models \langle\langle A \rangle\rangle \phi \mathcal{U} \phi'$  iff there is  $s_A \in \text{strat}(A)$  such that for all  $\lambda \in \text{out}(s_A, q)$  we have  $S, \lambda[i] \models \phi'$  and  $S, \lambda[j] \models \phi$  for some  $i \geq 0$  and for all  $0 \leq j < i$

Towards the statement that interpreting formulas over CGS and RCGS is equivalent (Theorem 1) we will describe a surjective translation function  $f$  translating each RCGS to a CGS. The following two lemmas will be useful in formulating the proof of Theorem 1.

The translation function  $f$  from RCGS to CGS is defined as follows:

$$f\langle \mathcal{A}, R, \mathcal{R}, Q, \Pi, \pi, \mathbb{A}, \delta \rangle = \langle \mathcal{A}, Q, \Pi, \pi, d, \delta' \rangle$$

where:

$$\begin{aligned} d_a(q) &= \mathbb{A}(q, r) && \text{where } a \in \mathcal{R}(q, r) \\ \delta'(q, \alpha_1, \dots, \alpha_n) &= \delta(q, v_1, \dots, v_{|R|}) && \text{where for each role } r \\ v_r &= \langle |\{i \in \mathcal{R}(q, r) \mid \alpha_i = 1\}|, \dots, |\{i \in \mathcal{R}(q, r) \mid \alpha_i = \mathbb{A}(q, r)\}| \rangle \end{aligned}$$

We describe a surjective function  $m : \text{strat}(f(S)) \rightarrow \text{strat}(S)$  mapping action tuples and strategies of  $f(S)$  to profiles and strategies of  $S$  respectively. For all  $A \subseteq \mathcal{A}$  and any action tuple for  $A$  at  $q$ ,

$t_q = \langle \alpha_{a_1}, \alpha_{a_2}, \dots, \alpha_{a_{|A|}} \rangle$  with  $1 \leq \alpha_{a_i} \leq d_{a_i}(q)$  for all  $1 \leq i \leq |A|$ , the  $A$ -profile  $m(t_q)$  is defined in the following way:

$$m(t_q) = \langle v(t_q, 1), \dots, v(t_q, |R|) \rangle \text{ where for all } 1 \leq r \leq |R| \text{ we have}$$

$$v(t_q, r) = \langle |\{a \in A_{r,q} \mid \alpha_a = 1\}|, \dots, |\{a \in A_{r,q} \mid \alpha_a = \mathbb{A}(q, r)\}| \rangle$$

**Lemma 1.** *For any RCGS  $S$  and any  $A \subseteq \mathcal{A}$ , the function  $m : \text{strat}(f(S), A) \rightarrow \text{strat}(S, A)$  is surjective.*

*Proof.* Let  $p_A$  be some strategy for  $A$  in  $S$ . We must show there is a strategy  $s_A$  in  $f(S)$  such that  $m(s_A) = p_A$ . For all  $q \in Q$ , we must define  $s_A(q)$  appropriately. Consider the profile  $p_A(q) = \langle v_1, \dots, v_{|R|} \rangle$  and note that by definition of a profile, all  $v_r$  for  $1 \leq r \leq |R|$  are  $A$ -votes for  $r$  and that by definition of an  $A$ -vote, we have  $\sum_{1 \leq i \leq \mathbb{A}(q,r)} v_r(i) = |A_{r,q}|$ . Also, for all agents  $a, a' \in A_{r,q}$  we know, by definition of  $f$ , that  $d_a(q) = d_{a'}(q) = \mathbb{A}(q, r)$ .

It follows that there are functions  $\alpha : A \rightarrow \mathbb{N}^+$  such that for all  $a \in A$ ,  $\alpha(a) \in [d_a(q)]$  and  $|\{a \in A_{r,q} \mid \alpha(a) = i\}| = v_r(i)$  for all  $1 \leq i \leq \mathbb{A}(q, r)$ , i.e.

$$v_r = \langle |\{a \in A_{r,q} \mid \alpha(a) = 1\}|, \dots, |\{a \in A_{r,q} \mid \alpha(a) = \mathbb{A}(q, r)\}| \rangle$$

We choose some such  $\alpha$  and  $s_A = \langle \alpha(a_1), \dots, \alpha(a_{|A|}) \rangle$ . Having defined  $s_A$  in this way, it is clear that  $m(s_A) = p_A$ .  $\square$

It will be useful to have access to the set of states that can result in the next step when  $A \subseteq \mathcal{A}$  follows strategy  $s_A$  at state  $q$ ,  $\text{succ}(q, s_A) = \{q' \in Q \mid \exists F \in \text{ext}(q, s_A) : \delta(q, F) = q'\}$ . Given either a CGS or an RCGS  $S$ , we define the set of sets of states that a coalition  $A$  can *enforce* in the next state of the game:

$$\text{force}(S, q, A) = \{\text{succ}(q, s_A) \mid s_A \text{ is a strategy for } A \text{ in } S\}.$$

Using the surjective function  $m$  we can prove the following lemma, showing that the “next time” strength of any coalition  $A$  is the same in  $S$  as it is in  $f(S)$ .

**Lemma 2.** *For any RCGS  $S$ , and state  $q \in Q$  and any coalition  $A \subseteq \mathcal{A}$ , we have  $\text{force}(S, A, q) = \text{force}(f(S), A, q)$ .*

*Proof.* By definition of *force* and Lemma 1 it is sufficient to show that for all  $s_A \in \text{strat}(f(S), A)$ , we have  $\text{succ}(S, m(s_A), q) = \text{succ}(f(S), s_A, q)$ . We show  $\subseteq$  as follows: Assume that  $q' \in \text{force}(S, m(s_A), q)$ . Then there is some complete profile  $P = \langle v_1, \dots, v_{|R|} \rangle$ , extending  $m(s_A)(q)$ , such that  $\delta(q, P) = q'$ . Let  $m(s_A)(q) = \langle w_1, \dots, w_{|R|} \rangle$  and form  $P' = \langle v'_1, \dots, v'_{|R|} \rangle$  defined by  $v'_i = v_i - w_i$  for all  $1 \leq i \leq |R|$ . Then each  $v'_i$  is an  $(\mathcal{A} \setminus A)$ -vote for role  $i$ , meaning that the sum of entries in the tuple  $v'_i$  is  $|( \mathcal{A} \setminus A )_{r,q}|$ . This means that we can define a function  $\alpha : \mathcal{A} \rightarrow \mathbb{N}^+$  such that for all  $a \in \mathcal{A}$ ,  $\alpha(a) \in [d_a(q)]$  and for all  $a \in A$ ,  $\alpha(a) = s_a(q)$  and for every  $r \in R$  and every  $a \in (\mathcal{A} \setminus A)$ , and every  $1 \leq j \leq \mathbb{A}(q, r)$ ,  $|\{a \in (\mathcal{A} \setminus A)_{r,q} \mid \alpha(a) = j\}| = v'_r(j)$ . Having defined  $\alpha$  like this it follows by definition of  $m$  that for all  $1 \leq j \leq \mathbb{A}(q, r)$ ,  $|\{a \in A_{r,q} \mid \alpha(a) = j\}| = w_r(j)$ . Then for all  $r \in R$  and all  $1 \leq j \leq \mathbb{A}(q, r)$  we have  $|\{a \in \mathcal{A}_{q,r} \mid \alpha(a) = j\}| = v_r(j)$ . By definition of  $f(S)$  it follows that  $q' = \delta(q, P) = \delta'(q, \alpha)$  so that  $q' \in \text{force}(f(S), s_A, q)$ . We conclude that  $\text{force}(S, f(s_A), q) \subseteq \text{force}(f(S), s_A, q)$ . The direction  $\supseteq$  follows easily from the definitions of  $m$  and  $f$ .  $\square$

We now state and prove the equivalence.

**Theorem 1.** *For any RCGS  $S$ , any  $\phi$  and any  $q \in Q$ , we have  $S, q \models \phi$  iff  $f(S), q \models_{\text{CGS}} \phi$ , where  $f$  is the surjective model-translation function.*

*Proof.* Given a structure  $S$ , and a formula  $\phi$ , we define  $true(S, \phi) = \{q \in Q \mid S, q \models \phi\}$ . Equivalence of models  $S$  and  $f(S)$  is now demonstrated by showing that the equivalence in next time strength established in Lemma 2 suffices to conclude that  $true(S, \phi) = true(f(S), \phi)$  for all  $\phi$ .

We prove the theorem by showing that for all  $\phi$ , we have  $true(S, \phi) = true(f(S), \phi)$ . We use induction on complexity of  $\phi$ . The base case for atomic formulas and the inductive steps for Boolean connectives are trivial, while the case of  $\langle\langle A \rangle\rangle \bigcirc \phi$  is a straightforward application of Lemma 2. For the cases of  $\langle\langle A \rangle\rangle \square \phi$  and  $\langle\langle A \rangle\rangle \phi \mathcal{U} \psi$  we rely on the following fixed point characterizations, which are well-known to hold for ATL, see for instance [4], and are also easily verified against definition 2:

$$\begin{aligned} \langle\langle A \rangle\rangle \square \phi &\leftrightarrow \phi \wedge \langle\langle A \rangle\rangle \bigcirc \langle\langle A \rangle\rangle \square \phi \\ \langle\langle A \rangle\rangle \phi_1 \mathcal{U} \phi_2 &\leftrightarrow \phi_2 \vee (\phi_1 \wedge \langle\langle A \rangle\rangle \bigcirc \langle\langle A \rangle\rangle \phi_1 \mathcal{U} \phi_2) \end{aligned} \quad (1)$$

We show the induction step for  $\langle\langle A \rangle\rangle \square \phi$ , taking as induction hypothesis  $true(S, \phi) = true(f(S), \phi)$ . The first equivalence above identifies  $Q' = true(S, \langle\langle A \rangle\rangle \square \phi)$  as the maximal subset of  $Q$  such that  $\phi$  is true at every state in  $Q'$  and such that  $A$  can enforce a state in  $Q'$  from every state in  $Q'$ , i.e. such that  $\forall q \in Q' : \exists Q'' \in force(q, A) : Q'' \subseteq Q'$ . Notice that a unique such set always exists. This is clear since the union of two sets satisfying the two requirements will itself satisfy them (possibly the empty set). The first requirement, namely that  $\phi$  is true at all states in  $Q'$ , holds for  $S$  iff it holds for  $f(S)$  by induction hypothesis. Lemma 2 states  $force(S, q, A) = force(f(S), q, A)$ , and this implies that also the second requirement holds in  $S$  iff it holds in  $f(S)$ . From this we conclude  $true(S, \langle\langle A \rangle\rangle \square \phi) = true(f(S), \langle\langle A \rangle\rangle \square \phi)$  as desired. The case for  $\langle\langle A \rangle\rangle \phi \mathcal{U} \psi$  is similar, using the second equivalence.  $\square$

**Example 2** (Sensor networks contd.). *To further illustrate the use of ATL interpreted over RCGS, we provide example formulas that are related to the structures shown in Example 1.*

*In the structure depicted in Figure 1, if at least  $k$  sensors signal something,  $p$  becomes true (e.g. the alarm is raised). This is expressed by formula  $\langle\langle A \rangle\rangle \bigcirc p$  which is satisfied in the structure from Figure 1, i.e.  $H_1, q_0 \models \langle\langle A \rangle\rangle \bigcirc p$  whenever  $|A \cap \mathcal{R}(q_0, 1)| \geq k$ . In Figure 2, the supervisor decides whether signals that indicate  $p$  are strong enough in order for him to signal  $q$ , e.g. raise the alarm. In this scenario, the sensors alone cannot raise the alarm, hence  $H_2, q_0 \not\models \langle\langle A \rangle\rangle \diamond q$  whenever  $A \cap \mathcal{R}(q_1, 2) = \emptyset$  (which means that whenever the coalition  $A$  does not include the supervisor,  $q$  cannot be enforced). On the other hand,  $H_2, q_0 \models \langle\langle A \rangle\rangle \bigcirc \langle\langle B \rangle\rangle \bigcirc q$  whenever  $|A \cap \mathcal{R}(q_0, 1)| \geq k$  and  $B \cap \mathcal{R}(q_1, 2) \neq \emptyset$  (which means that the coalition of agents without a supervisor can enable the supervisor to take action).*

### 3 Model checking and the size of models

In this section we will see how using roles can lead to a dramatic decrease in the size of ATL models. We first investigate the size of models in terms of the number of roles, players and actions, and then we analyze model checking of ATL over concurrent game structures with roles.

Given a set of numbers  $[a]$  and a number  $n$ , it is a well-known combinatorial fact that the number of ways in which to choose  $n$  elements from  $[a]$ , allowing repetitions, is  $\frac{(n+(a-1))!}{n!(a-1)!}$ . Furthermore, this number satisfies the following two inequalities:<sup>1</sup>

$$\frac{(n+(a-1))!}{n!(a-1)!} \leq a^n \quad \text{and} \quad \frac{(n+(a-1))!}{n!(a-1)!} \leq n^a. \quad (2)$$

<sup>1</sup>If this is not clear, remember that  $n^a$  and  $a^n$  are the number of functions  $[n]^{[a]}$  and  $[a]^{[n]}$  respectively. It should not be hard to see that all ways in which to choose  $n$  elements from  $a$  induce non-intersecting sets of functions of both types.

These two inequalities provide us with an upper bound on the *size* of RCGS models that makes it easy to compare their sizes to that of CGS models. Typically, the size of concurrent game structures is dominated by the size of the domain of the transition function. For an RCGS and a given state  $q \in Q$  this is the number of complete profiles at  $q$ . To measure it, remember that every complete profile is an  $|R|$ -tuple of votes  $v_r$ , one for each role  $r \in R$ . Also remember that a vote  $v_r$  for  $r \in R$  is an  $\mathbb{A}(q, r)$ -tuple such that the sum of entries is  $|\mathcal{A}_{q,r}|$ . Equivalently, the vote  $v_r$  can be seen as the number of ways in which we can make  $|\mathcal{A}_{q,r}|$  choices, allowing repetitions, from a set of  $\mathbb{A}(q, r)$  alternatives. Looking at it this way, we obtain:

$$|P(q)| = \prod_{r \in R} \frac{(|\mathcal{A}_{q,r}| + (\mathbb{A}(q, r) - 1))!}{|\mathcal{A}_{q,r}|! (\mathbb{A}(q, r) - 1)!}.$$

We sum over all  $q \in Q$  to obtain what we consider to be the size of an RCGS  $S$ . In light of Equation 2, it follows that the size of  $S$  is upper bounded by both of the following expressions.

$$\mathcal{O}(\sum_{q \in Q} \prod_{r \in R} |\mathcal{A}_{q,r}|^{\mathbb{A}(q,r)}) \quad \text{and} \quad \mathcal{O}(\sum_{q \in Q} \prod_{r \in R} \mathbb{A}(q,r)^{|\mathcal{A}_{q,r}|}). \quad (3)$$

We observe that growth in the size of models is polynomial in  $a = \max_{q \in Q, r \in R} \mathbb{A}(q, r)$  if  $n = |\mathcal{A}|$  and  $|R|$  is fixed, and polynomial in  $p = \max_{q \in Q, r \in R} |\mathcal{A}_{q,r}|$  if  $a$  and  $|R|$  are fixed. This identifies a significant potential advantage arising from introducing roles to the semantics of ATL. The size of a CGS  $M$ , when measured in the same way, replacing complete profiles at  $q$  by complete action tuples at  $q$ , grows exponentially in the players whenever the players have more than one action. We stress that we are *not* just counting the number of transitions in our models differently. We do have an additional parameter, the roles, but this is a new semantic construct that gives rise to genuinely different semantic structures. We have established that it is possible to use them to give the semantics of ATL, but this does not mean that there is not more to be said about them. Particularly crucial is the question of model checking over RCGS models.

### 3.1 Model checking using roles

For ATL there is a well known model checking algorithm [1]. It does model checking in time linear in the length of the formula and the size of the model. Given a structure  $S$ , and a formula  $\phi$ , the standard model checking algorithm  $mcheck(S, \phi)$  returns the set of states of  $S$  where  $\phi$  holds.

The algorithm depends on a function  $enforce(S, A, q, Q')$ , which given a structure  $S$ , a coalition  $A$ , a state  $q \in Q$  and a set of states  $Q'$  answers true or false depending on whether or not  $A$  can enforce  $Q'$  from  $q$ . This is the only part of the standard algorithm that needs to be modified to accommodate roles.

For all profiles  $F \in P(q, A)$  the  $enforce$  algorithm runs through all complete profiles  $F' \in P(q)$  that extend  $F$ . It is polynomial in the number of complete profiles, since for any coalition  $A$  and state  $q$  we have  $|P(q, A)| \leq |P(q)|$ , meaning that the complexity of  $enforce$  is upper bounded by  $|P(q)|^2$ . Given a fixed length formula and a fixed number of states,  $enforce$  dominates the running time of  $mcheck$ . It follows that model checking of ATL over concurrent game structures with roles is polynomial in the size of the model. We summarize this result.

**Proposition 1.** *Given a CGS  $S$  and a formula  $\phi$ ,  $mcheck(S, \phi)$  takes time  $\mathcal{O}(le^2)$  where  $l$  is the length of  $\phi$  and  $e = \sum_{q \in Q} |P(q)|$  is the total number of transitions in  $S$*

```

for  $F \in P(q, A)$  do
   $p \leftarrow true$ 
  for  $F' \in ext(q, F)$  do
    if  $\delta(q, F') \notin Q'$  then
       $p \leftarrow false$ 
  if  $p = true$  then
    return  $true$ 
return  $false$ 

```

Figure 4:  $enforce(S, A, q, Q')$

Since model checking ATL over CGSs takes only linear time,  $\mathcal{O}(le)$ , adding roles apparently makes model checking harder. On the other hand, the *size* of CGS models can be bigger by an exponential factor, making model checking much easier after adding roles. In light of the bounds we have on the size of models, c.f. Equation 3, we find that as long as the roles and the actions remain fixed, complexity of model checking is only polynomial in the number of agents. This is a potentially significant argument in favor of including roles in the semantics.

Roles should be used at the modeling stage, as they give the modeler an opportunity for exploiting homogeneity of the system under consideration. We think that it is reasonable to hypothesize that in practice, most large scale multi-agent systems that lend themselves well to modeling by ATL exhibit significant homogeneity.

The question arises as to whether or not using an RCGS is *always* the best choice, or if there are situations when the losses incurred in the complexity of model checking outweigh the gains we make in terms of the size of models. We conclude with the following proposition, also shown in [2], which states that as long we use the standard algorithm, model checking any CGS  $M$  can be done at least as quickly by model checking an *arbitrary*  $S \in f^-(M)$ .

**Proposition 2.** *Given any CGS-model  $M$  and any formula  $\phi$ , let  $c(mcheck(M, \phi))$  denote the complexity of running  $mcheck(M, \phi)$ . We have, for all  $S \in f^-(M)$ , that complexity of running  $mcheck(S, \phi)$  is  $\mathcal{O}(c(mcheck(M, \phi)))$*

*Proof.* It is clear that for any  $S \in f^-(M)$ , running  $mcheck(S, \phi)$  and  $mcheck(M, \phi)$ , a difference in overall complexity can arise only from a difference in the complexity of *enforce*. So we compare the complexity of  $enforce(S, A, q, Q')$  and  $enforce(M, A, q, Q')$  for some arbitrary  $q \in Q$ ,  $Q' \subseteq Q$ . The complexity in both cases involves passing through all complete extensions of all strategies for  $A$  at  $q$ . The sizes of these sets can be compared as follows, the first inequality is an instance of Equation 2 and the equalities follow from definition of  $f$  and the fact that  $M = f(S)$ .

$$\begin{aligned}
\prod_{r \in R} \left( \frac{(|A_{r,q}| + (\mathbb{A}(r, q) - 1))!}{|A_{r,q}|! (\mathbb{A}(r, q) - 1)!} \right) &\times \prod_{r \in R} \left( \frac{((|\mathcal{A}_{q,r}| - |A_{r,q}|) + (\mathbb{A}(r, q) - 1))!}{(|\mathcal{A}_{q,r}| - |A_{r,q}|)! (\mathbb{A}(r, q) - 1)!} \right) \\
&\leq \left( \prod_{r \in R} \mathbb{A}(r, q)^{|A_{r,q}|} \times \prod_{r \in R} \mathbb{A}(r, q)^{|\mathcal{A}_{q,r}| - |A_{r,q}|} \right) \\
&= \prod_{r \in R} \left( \prod_{a \in A_{r,q}} \mathbb{A}(r, q) \right) \times \prod_{r \in R} \left( \prod_{a \in \mathcal{A}_{a,r} \setminus A_{r,q}} \mathbb{A}(r, q) \right) \\
&= \left( \prod_{a \in A} d_a(q) \times \prod_{a \in \mathcal{A} \setminus A} d_a(q) \right) = \prod_{a \in \mathcal{A}} d_a(q)
\end{aligned}$$

We started with the number of profiles (transitions) we need to inspect when running *enforce* on  $S$  at  $q$ , and ended with the number of action tuples (transitions) we need to inspect when running *enforce* on  $M = f(S)$ . Since we showed the first to be smaller or equal to the latter and the execution of all other elements of *mcheck* are identical between  $S$  and  $M$ , the claim follows.  $\square$



## 4 Conclusions, related and future work

In this paper we have described a new type of semantics for the strategic logic ATL. We have provided illustrating examples and argued that although in principle model checking ATL interpreted over concurrent game structures with roles is harder than the standard approach, it is still polynomial and can generate exponentially smaller models. We believe this provides evidence that concurrent game structures with roles are an interesting semantics for ATL, and should be investigated further.

Relating our work to ideas already present in the literature we find it somewhat similar to the idea of exploiting symmetry in model checking, as investigated by Sistla and Godefroid [7]. However, our approach is different, since we look at agent symmetries in ATL as the basis of a new semantics. When it comes to work related directly to strategic logics, we find no similar ideas present, hence concluding that our approach is indeed novel.

For future work we plan on investigating the homogeneous aspect of our ‘roles’ in more depth. We are currently working on a derivative of ATL with a different language that will fully exploit the role based semantics.

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